

σ , κ , $f_0(980)$ and $a_0(980)$

D.V. Bugg

Queen Mary, University of London, Mile End Rd., London E1 4NS, UK

Abstract

Both σ and κ are well established from E791 data on $D \rightarrow 3\pi$ and $K\pi\pi$ and BES II data on $J/\Psi \rightarrow \omega\pi^+\pi^-$ and $K^+K^-\pi^+\pi^-$. These fits are accurately consistent with $\pi\pi$ and $K\pi$ elastic scattering when one allows for the Adler zero which arises from Chiral Symmetry Breaking. The phase variation with mass is consistent between elastic scattering and production data. Also Colangelo et al. show that crossing symmetry and dispersion relations for $\pi\pi$ elastic scattering demand a σ pole within 2 standard deviations of the pole fitted to production data. Oset and collaborators find similar results using unitarised Chiral Perturbation Theory. Possible interpretations of σ , κ , $f_0(980)$ and $a_0(980)$ are explored.

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1 Introduction

This is an extended version of a report for the Proceedings of the Hadron05 conference, where space did not allow full discussion. An extensive collection of figures of data is given here. Also technical points of discrepancies between analyses are discussed in detail. A section involving these technical points is denoted by an asterix.

This work has been assisted greatly by a working group on Scalars, established at Hadron05. I am grateful to the large number of contributors to this working group. Opinions have frequently differed, so the responsibility is mine for trying to arrive at a consensus; at several points I need to explore conflicts of opinion and sometimes try to reach a conclusion.

2 The σ pole

Early evidence for the σ pole arose from elastic scattering data. Markushin and Locher [1] summarise many determinations. Renewed interest was sparked off by E791 data on $D^+ \rightarrow (\pi^+\pi^-)\pi^+$ [2]. The $\pi\pi$ mass projection, shown in Fig. 1, has a low mass peak which was fitted by a pole shown in the first entry of Table 1 below. Their fit assumed a Breit-Wigner resonance with $\Gamma(s) \propto \rho(s)$, where $\rho(s)$ is Lorentz invariant phase space $2k/\sqrt{s} = \sqrt{1 - 4m_\pi^2/s}$ and k is centre of mass momentum. This choice for $\Gamma(s)$ will later be shown to be inappropriate, but correcting it to a better form introduces only changes of detail. Oller has refitted the data in a way consistent with Chiral Perturbation Theory; there is some increase in the width of the pole [3].

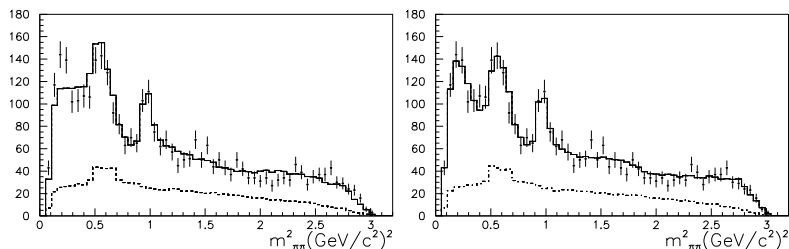


Figure 1: The $\pi\pi$ mass projection of E791 data for $D^+ \rightarrow \pi^+(\pi^-\pi^+)$, (a) without, (b) with σ in the fit.

Higher statistics data from BES II [4] on $J/\Psi \rightarrow \omega\pi^+\pi^-$ are shown in Fig. 2. They are dominated by $f_2(1270)$, $b_1(1235)$ and σ , which is clearly visible as a flat band along the upper right-hand edge of the Dalitz plot in (b). The 0^+ contribution is shown in (e). There is a marginal 2 standard deviation contribution from $f_0(980)$. In order to test the sensitivity of the extrapolation to the pole, four types of parametrisation were tried. All are consistent with an average pole position $M = (541 \pm 39) - i(252 \pm 42)$ MeV.

3 How to parametrise the σ

Fig. 3(a) shows the intensity of $\pi\pi$ elastic scattering v. mass. Why is there no low mass peak like that in production data of Fig. 2? The explanation was

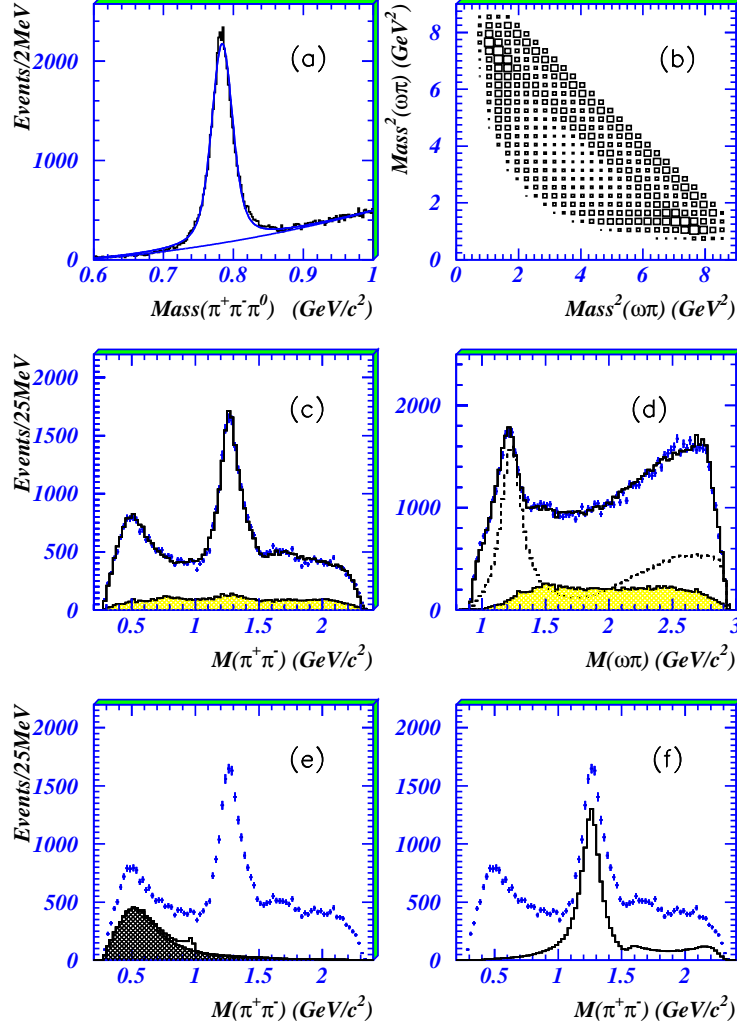


Figure 2: BES II data for $J/\Psi \rightarrow \omega\pi^+\pi^-$. (a) The ω mass peak showing background from $\pi^+\pi^-\pi^+\pi^-\pi^0$; (b) Dalitz plot; (c) $\pi\pi$ mass projection; the histogram shows the fit and the hatched area the experimental background; (d) $\omega\pi$ mass projection; the dashed histogram shows the $b_1(1235)\pi$ contribution (two combinations); (e) the σ contribution (hatched) and the full 0^+ contribution including $f_0(980)$; (f) 2^+ contribution.

given in 1965-6 by Adler and Weinberg [5]. They proposed that massless π of zero momentum have zero elastic amplitude. If the $I = 0$ S-wave $\pi\pi \rightarrow \pi\pi$ amplitude is expanded as a power series $f = am_\pi^2 + bk^2$, consistency between s , t and u channels requires $f \propto (s - 0.5m_\pi^2)$ and a zero at the Adler point $s_A = 0.5m_\pi^2$. Fig. 3(b) shows the result of dividing Fig. 3(a) by $(s - s_A)^2$. Instantly one sees a resemblance with the σ peak of Fig. 2. So the solution to the puzzle is that the matrix element for $\pi\pi$ elastic scattering is strongly s -dependent: a situation unlike most other resonances.

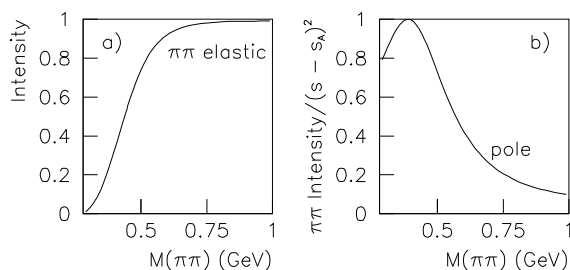


Figure 3: (a) The $\pi\pi$ intensity in elastic scattering, (b) with the Adler zero divided out.

Let us write the elastic σ amplitude as

$$\begin{aligned} T_{el}^{00} &= (\eta \exp(2i\delta) - 1)/2i \\ &= \frac{N(s)}{D(s)} = \frac{N_{el}(s)}{M^2 - s - iN_{tot}(s)}. \end{aligned} \quad (1)$$

Here $N_{el}(s)$ is real for $s \geq 0$. The phase variation comes purely from $D(s)$; this denominator is universal for all processes involving a $\pi\pi$ pair. [That is the assumption on which the Particle Data Tables are based]. For elastic scattering, the Adler zero in $N(s)$ nearly cancels the σ pole for low masses. However, the numerator $N(s)$ is not universal; it is quite different for production processes, where the left-hand cut is distant. Later, it will be shown that E791 data for $D^+ \rightarrow (K^-\pi^+)\pi^+$ require $N(s)_{prod} = 1$ within errors. The production amplitude will therefore be written

$$T_{prod}^{00} = \Lambda/D(s), \quad (2)$$

where Λ is a complex coupling constant.

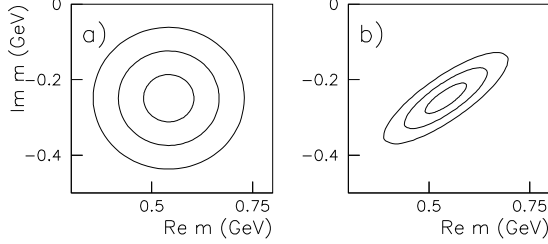


Figure 4: Contours of intensity for (a) production, (b) elastic scattering.

Fig. 4 sketches contours of constant intensity for (a) production, (b) elastic scattering. In the latter case, the effect of the Adler zero is to suppress the intensity near $s = 0$. The elastic phase shift on the real s axis (where experiments are done) reaches 90° only at $M > 900$ MeV, far above the position of the pole. It is this feature which confuses many people. However, the phase varies rapidly off the real axis because the width increases with s and because of consequent effects of analyticity.

A simultaneous fit is made to BES II data, K_{e4} [6] and Cern-Munich data [7] using the empirical form:

$$N(s) = M(s - 0.5m_\pi^2) \exp[-(s - M^2)/A](1 + \beta s)\rho_{\pi\pi}(s) + M\Gamma_{4\pi}(s). \quad (3)$$

The exponential is required by $\pi\pi$ elastic data to cut off $N(s)$ above 1 GeV. The term $M\Gamma_{4\pi}$ accounts for 4π inelasticity above ~ 1 GeV, but has little effect on the σ pole. Note from Fig. 2(e) that the σ intensity fitted to BES data falls to a low value above 1 GeV. The σ pole is therefore distinct from the broad $f_0(1535)$ fitted by Anisovich et al. [8]. It is also distinct from the broad pole fitted around 1 GeV by Au, Morgan and Pennington [9].

Below the KK threshold, the elastic amplitude must follow the unitarity circle. There are small contributions from the low mass tails of $f_0(1370)$, $f_0(1500)$, etc. and a contribution from $f_0(980)$ which is large around 1 GeV. These are included by writing

$$T_{\pi\pi}^{00} = (S_\sigma S_{980} S_{1370} S_{1500} - 1)/(2i), \quad (4)$$

where S is the S -matrix $\eta \exp(2i\delta) = 2i(1+T)$ for each individual resonance. Below the KK threshold, $\eta = 1$ for all amplitudes, so phases due to individual resonances add.

For production, there are hundreds of open channels for D and J/Ψ decays. Within individual channels, unitarity plays a negligible role. In the standard isobar model, *amplitudes* are added using a complex coupling constant $\Lambda = g \exp(i\phi_0)$ for each amplitude; the phase ϕ_0 is constant over the whole Dalitz plot. For $J/\Psi \rightarrow \omega\sigma$, ϕ_0 is the phase of $\sigma\omega$ elastic scattering at the J/Ψ mass; it is unknown, so g and ϕ_0 need to be fitted to the data.

The K_{e4} data are available up to 380 MeV and there is then a gap in elastic scattering data until 560 MeV, where Cern-Munich data begin. The σ pole lies in the mass range where there are no elastic data. Although this gap may be bridged by using dispersion relations, the production data are obviously important in filling the gap directly.

4 The phase of the σ

In the BES data, the $b_1\pi$ channel contributes 41% intensity and $\sigma\omega$ 19%. There are strong interferences between them which determine the phase variation of the σ with mass. The data have been divided into bins 100 MeV wide from 400 to 1000 MeV. Fig. 5 shows phases for individual bins, keeping magnitudes fixed at values from the global fit: this achieves optimum accuracy, since there is noise in the magnitude in individual bins if it is set free. The full curve shows the phase from elastic scattering data. The agreement with the bin-by-bin fit shows that the Breit-Wigner form of eqn. (1) indeed accounts for the phase in all data. The implication is that the data at low mass may be described by a single resonance, except for well understood contributions from $f_0(980)$, $f_0(1370)$ and $f_0(1500)$. It is *not* necessary to include a background amplitude. Another way of stating this is that if any background is present, it is the same in both elastic scattering and production and may be absorbed algebraically into the parametrisation of the σ amplitude.

Within the scalar working group, it has been suggested that phases could be modified by rescattering of pions from σ decay off the ω ; although this must happen at some level, its contribution seems to be below the level of experimental errors at present.

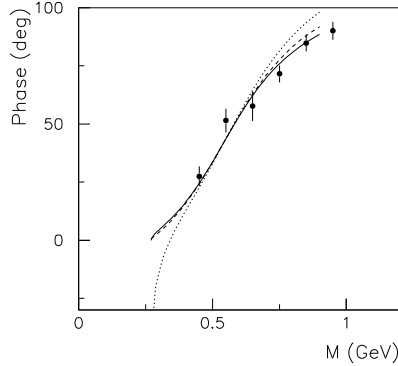


Figure 5: The phase of the σ amplitude in mass bins 100 MeV wide, compared with the global fit (full curve), a Breit-Wigner of constant width (dashed) and a Breit-Wigner with $\Gamma \propto \rho_{\pi\pi}$ (dotted).

The dotted curve of Fig. 5 shows the phase variation if the amplitude for the σ is expressed in the form $\Gamma \propto \rho_{\pi\pi}$. This gives a marginally poorer fit, but cannot be distinguished cleanly using the BES data. Unfortunately, the b_1 band runs off the corner of the Dalitz plot and does not interfere significantly with the σ below 400 MeV.

5 Theory

It is important to realise that there is a large background of theoretical work on elastic scattering and related processes like $\pi\pi \rightarrow KK$.

Colangelo, Gasser and Leutwyler [10] have made a precise determination of the σ pole from elastic scattering and K_{e4} data without using production data. They use crossing symmetry to calculate the amplitude on the left-hand cut ($s < 0$) from amplitudes at $s > 4m_\pi^2$ (right-hand cut). For a given s and t , the two are related by a simple isospin matrix; the $\pi\pi$ S-wave amplitudes is then evaluated from an integration over the appropriate range of t (and likewise for P and D-waves, including appropriate Legendre polynomials). As a further constraint, they use fixed t dispersion relations (Roy equations).

State	Reference	Data	Pole position (MeV)
σ	[2]	$D^+ \rightarrow (\pi^+\pi^-)\pi^+$	$(489 \pm 26) - i(173 \pm 26)$
	[3]	$D^+ \rightarrow (\pi^+\pi^-)\pi^+$	$470 - i220$
	[4]	$J/\Psi \rightarrow \omega(\pi^+\pi^-)$	$(541 \pm 39) - i(252 \pm 42)$
	[10]	$\pi\pi \rightarrow \pi\pi$	$(470 \pm 30) - i(295 \pm 20)$
	[13]	$\pi\pi \rightarrow \pi\pi$	$445 - i221$
	[25]	$\pi\pi \rightarrow \pi\pi$	$(470 \pm 50) - i(285 \pm 50)$
	[27]	$D^+ \rightarrow (\pi^+\pi^-)\pi^+$	$(455 \pm 36) - i(190 \pm 36)$
κ	[30]	$D^+ \rightarrow (K^-\pi^+)\pi^+$	$(721 \pm 61) - i(292 \pm 131)$
	[32]	$J/\Psi \rightarrow K^+\pi^-K^-\pi^+$	$(760 \pm 41) - i(420 \pm 75)$
	[31]	$J/\Psi \rightarrow K^+\pi^-K^-\pi^+$	$(841 \pm 82) - i(309 \pm 87)$
	[36]	$K\pi \rightarrow K\pi$	$(722 \pm 60) - i(386 \pm 50)$
	here	all	$750^{+30}_{-55} - i(342 \pm 60)$
	[3]	$D^+ \rightarrow (K^-\pi^+)\pi^+$	$710 - i310$
	[12]	$K\pi \rightarrow K\pi$	$(770 - i(250 - 425))$
	[14]	$K\pi \rightarrow K\pi$	$(708 - i305)$
	[15]	$K\pi \rightarrow K\pi$	$(753 - i235)$
	[24]	$K\pi \rightarrow K\pi$	$(594 \pm 79) - i(362 \pm 322)$
$f_0(980)$	[37]	$J/\Psi \rightarrow \phi\pi^+\pi^-$	$(998 \pm 4) - i(17 \pm 4)$
	[12]	$\pi\pi \rightarrow \pi\pi$ and KK	$994 - i14$
$a_0(980)$	[39]	$\bar{p}p \rightarrow \eta\pi\pi$ and $\omega\eta\pi^0$	$(1036 \pm 5) - i(84 \pm 9)$

Table 1: Summary of pole positions.

This extends greatly the range of s in which the S-wave is known and fixes the $I = 0$ and 2 scattering lengths accurately. In this work, $Re T_{\pi\pi}^{00}$ is accurately determined and contains a very clear Adler zero at $s = 0.45m_{\pi\pi}^2$ (displaced by a tiny amount from Weinberg's prediction because of higher powers of k). The published work fits $\pi\pi$ masses up to 800 MeV and demands a σ pole at $M = (470 \pm 30) - i(295 \pm 20)$ MeV, i.e. about 2 standard deviations from the BES experimental value. Further work is in progress, to extend the fitted mass range to 1150 MeV, above $f_0(980)$ and the KK threshold.

In a series of papers [11-16], Oset, Oller, Pelaez and collaborators have fitted data using 'unitarised' Chiral Perturbation Theory. In outline, they

use ChPT for lowest order and take rescattering from the next order. They successfully fit not only the $I = 0$ S-wave, but also reproduce the repulsive $I = 2$ S-wave. Their pole positions are given in Table 1.

Schechter's group has also contributed a series of papers on all of σ , κ , $f_0(980)$ and $a_0(980)$ [17–21]. This work examines possible mixing between 2-quark and 4-quark states.

Zheng and collaborators at Peking University have developed new types of dispersion relations and have applied them to analysis of data on the σ and κ [22–25]. In particular, they clarify how unphysical sub-threshold poles can originate from the $1/\sqrt{s}$ dependence of phase space factors; they also show that a pole is possible without the phase shift reaching 90° , as may well happen in $K\pi$ scattering. Thirdly, in Ref. 25, they repeat the analysis of Colangelo et al. with different dispersion relations fitting the left-hand cut; they confirm the necessity for the σ pole.

6 K-matrix fits *

Focus data on $D^+ \rightarrow 3\pi$ exhibit a similar low mass peak to E791 data [26]. If it is fitted with a Breit-Wigner with $\Gamma \propto \rho_{\pi\pi}$, there is a pole at $M = (455 \pm 36) - i(190 \pm 36)$ MeV [27]. However, Focus show that their data can also be fitted with a K-matrix parametrisation of Anisovich and Sarantsev [28] which does not include a σ pole. This provides an escape route for those who do not wish to believe in the σ pole. The Babar collaboration has likewise found that their data can be fitted by the same K-matrix parametrisation [29]. This point requires some detailed and (unfortunately) critical comments.

There are several problems. The K-matrix parametrisation includes low mass poles at ~ 600 and 1200 MeV and at $s = -3$ GeV². Focus report their parametrisation fully, so it is straightforward to follow details of the fit.

The first point is that the low mass region is being fitted by 3 poles instead of 1. This introduces large flexibility into the fit.

The second point is that the pole below threshold plays a major role, but is questionable. If it is a zero of $D(s)$, it is a very deeply bound state which is more questionable than the σ . A more likely interpretation is a pole of $N(s)$. However, the N function is different for $\pi\pi$ elastic scattering and production reactions. So there is no reason to think that a pole fitted to elastic scattering will be appropriate to production. Furthermore, the N

function is normally interpreted as providing driving forces which generate the amplitudes on the right-hand cut, e.g. resonances. Including them as specific poles in searching for resonances looks like double-counting.

There is a third problem. In the K-matrix approach, $\pi\pi$ elastic scattering below the KK threshold is fitted not only by K_{11} (i.e. $\pi\pi \rightarrow \pi\pi$) but also by the analytic continuation of K_{12} below the KK threshold. In the case of D decays, this corresponds to $D \rightarrow (KK)\pi$, followed by KK rescattering to $\pi\pi$. This component should be constrained to reproduce data on $D \rightarrow KK\pi$ above threshold; however, that has not been done.

In K-matrix fits to $\pi\pi$ elastic scattering below the KK threshold, I know from personal experience that there is large cross-talk between K_{11} and K_{12} . K_{12} increases below threshold as $|k|/\sqrt{s}$ where $|k|$ is the magnitude of the virtual momentum in the KK channel. The increasing K_{12} as $s \rightarrow 0$ looks not unlike a σ pole. The extrapolation below KK threshold is reliable only close to threshold. In the Flatté formula for $f_0(980)$, as an example, it plays an important role in contributing to the real part of the amplitude near resonance. However, far from threshold the extrapolation is hazardous. This is a general problem which has been seen elsewhere. For example, in calculations of charmonium levels, open charm states with large decay widths perturb energy levels by major amounts.

My view is that there is a stabilising factor which needs to be brought into play. It is common experience that form factors play a role above threshold with a radius of order 0.6–0.8 fm. If such a form factor is needed above threshold, the analytic continuation below threshold breaks down because it depends critically on the imaginary part of the amplitude extending to infinity. With a form factor, the continuation below threshold can instead be done using a dispersion integral:

$$Re K_{12}(s) = \frac{1}{\pi} \int \frac{Im K_{12}(s') ds'}{s' - s} \quad (5)$$

I have carried out calculations along these lines. The form factor gives $Im K_{12}$ a localised peak close to threshold and the result is that $Re K_{12}$ drops fairly rapidly below threshold. In practice, it falls below threshold with a form similar to the fall-off above threshold, e.g. $\exp(-\alpha|k|^2)$. This behaviour is totally different from the analytic continuation without a form factor, and can easily lead to an order of magnitude reduction in $Re K_{12}$ by the time one reaches the position of the σ pole. Focus show in their Figs. 7 and 8, the

magnitudes they fit to sub-threshold KK , 4π , $\eta\eta$ and $\eta\eta'$ amplitudes; all are very large compared with fitted values of these amplitudes above threshold, typically by one to two orders of magnitude. It seems physically unreasonable that channels like $\eta\eta$ and $\eta\eta'$, which are quite small in the physical region, should make large sub-threshold contributions.

Putting this point in a different way, why stop at the 4π channel above threshold? Why not include an infinity of open channels at high mass: 6π , 8π , $\dots 100\pi \dots$. This leads to the possibility of fitting *all* mesons resonances as interferences between an infinity of distant high-mass singularities. This contradicts the conventional view that low mass peaks may be attributed to nearby singularities unless there is a specific convincing alternative.

My impression is that the K-matrix is dependable above threshold, where it is fitted directly to data on the inelastic channels. But below threshold, it introduces excessive flexibility. What is urgently needed is to constrain it to fit the left-hand cut as well, following the work of Colangelo et al.

Conversely, a criticism of published fits based on the T-matrix is that they ignore sub-threshold contributions from 4π , KK and $\eta\eta$. A more rigorous expression for the Breit-Wigner denominator of eqn. (1) is

$$D(s) = M^2 - s - m(s) - iN_{tot}(s) \quad (6)$$

$$m(s) = \frac{M^2 - s}{\pi} \int \frac{N_{tot}(s')ds'}{(s' - s)(M^2 - s')}. \quad (7)$$

In work not yet published, I have used this full form. It takes account of the opening of the inelastic channels and also the effect of the dispersive contribution in eqn. (3) to $Re K_{11}$. The detailed parameters of the fit change, but effects on the position of the σ pole are within the presently quoted errors. The essential reason for this is that the BES data define the position and width of the peak unambiguously. What happens is that the additional term $m(s)$ in the denominator is accommodated by corresponding changes in the already flexible form of $N(s)$.

7 The κ pole

E791 data on $D^+ \rightarrow (K^-\pi^+)\pi^+$ provided the first evidence for the κ pole from production data [30]. Their parametrisation gives a pole at $M = (721 \pm$

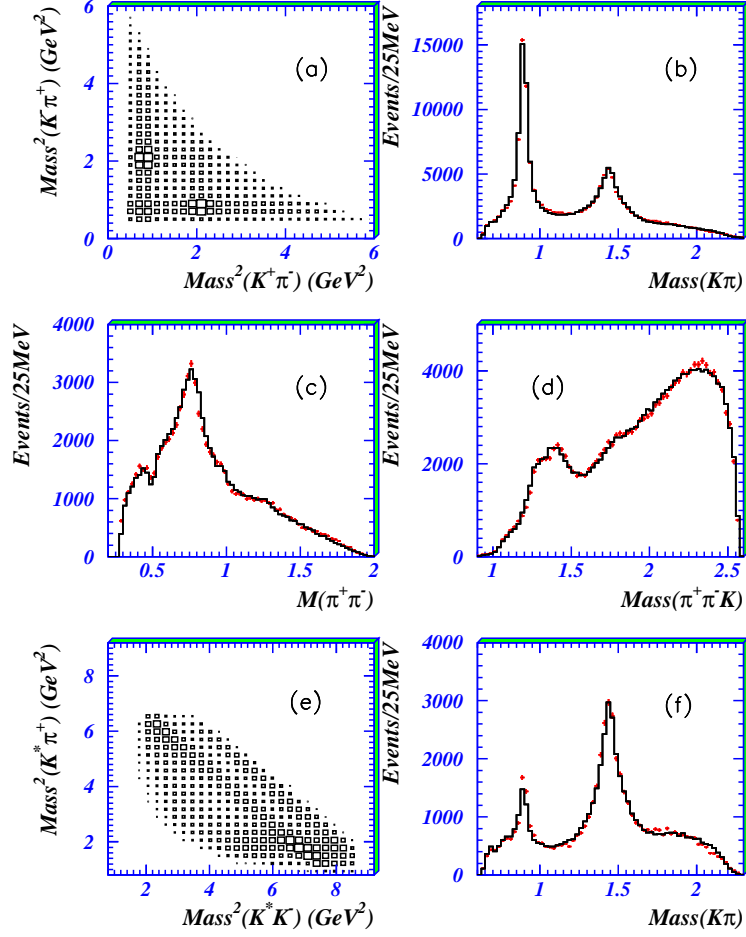


Figure 6: BES II data for $J/\Psi \rightarrow K^+K^-\pi^+\pi^-$. (a) The scatter plot $M(K^+\pi^-)$ v. $M(K^-\pi^+)$. Projections on to (b) $K^\pm\pi^\mp$ mass, (c) and (d) $\pi\pi$ and $\pi\pi K$ mass. (e) The Dalitz plot for events where one $K^\pm\pi^\mp$ pair has mass 892 ± 100 MeV. (f) Mass projection of the second $K^\mp\pi^\pm$ pair for the same selection as (e).

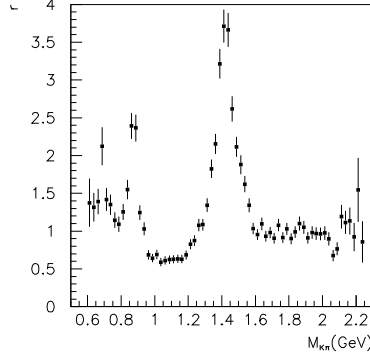


Figure 7: (a) The $K\pi$ mass projection of Fig. 6(f) divided by $K\pi$ phase space, in bins of 20 MeV.

$61) - i(292 \pm 131)$ MeV. A combined fit to these and other data will be shown below.

Next, BES II data on $J/\Psi \rightarrow K^+\pi^-K^-\pi^+$ provide further evidence for the κ in the channel $J/\Psi \rightarrow K^*(890)\kappa$ [31,32]. The scatter plot and mass projections are shown in Fig. 6; histograms show the fit. There are clear peaks due to $K^*(890)$, $K_{0,2}(1430)$, $\rho(770)$, $\rho(1270)$, $K_1(1270 + 1400)$ and $K_1(1770)$. If one selects $K^\pm\pi^\mp$ pairs within 50 MeV of 892 MeV, the mass projection of the other $K^\mp\pi^\pm$ pair is shown in panel (f). A broad $K\pi$ S-wave component is visible under the $K^*(890)$. Fig. 7 shows the effect of factoring out $K\pi$ phase space in the 4-body system. There is a broad peak below 750 MeV, which is fitted as the κ signal.

Data over the whole of phase space are fitted. Full details are given in Ref. [32]. The fit is made simultaneously to BES data and LASS data [33] on the $K\pi$ $I = 1/2$ S-wave. Eqn. (1) is used with

$$N(s) = M(s - s_A) \exp(-\alpha\sqrt{s})\rho_{K\pi}(s) \quad (8)$$

with $s_A = m_K^2 - 0.5m_\pi^2$. [Fits of similar quality may be obtained by replacing $\exp(-\alpha\sqrt{s})$ with $\exp(-\alpha's)$ or $1/(s - s_0)$ with marginally poorer log likelihood]. If the factor $(s - s_A)$ is omitted from eqn. (8), the poor fit is shown in Fig. 8(a). The κ mass projection is shown by the full histogram of

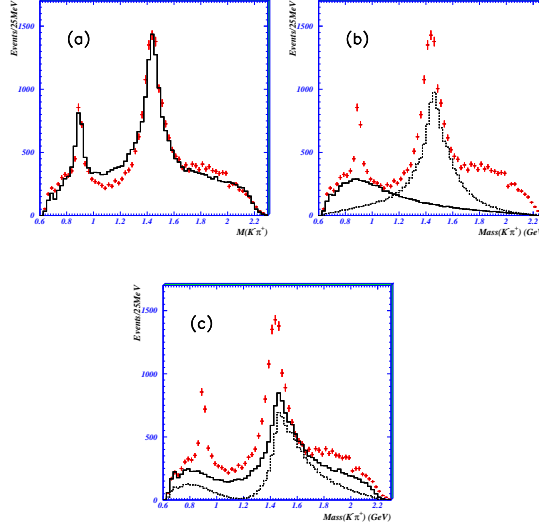


Figure 8: (a) The poor fit when $K^*(890)\kappa$ is removed; (b) individual contributions from κ (full histogram) and $K_0(1430)$ (dashed); (c) the coherent sum of $\kappa + K_0(1430)$ (dashed histogram) and the coherent sum $\kappa + K_0(1430) + K_1(1270) + K_1(1400)$ (full).

Fig. 8(b) and the $K_0(1430)$ mass projection as the dashed histogram. There is destructive interference between κ and $K_0(1430)$; their coherent sum is shown by the dashed histogram in Fig. 8(c). Sensitivity to this interference is one reason for fitting the LASS data simultaneously; it is absent in those data, where the phases of the two components add. A second reason is to examine whether eqn. (8) fits both sets of data successfully.

Since K_1 decays populate the low mass $K\pi$ range, it is essential to demonstrate that K_1 decays do not reproduce the κ peak. Omitting the κ leads to a fit worse by > 1000 in log likelihood.

An essential feature of the analysis is the separation of the 1430 MeV $K\pi$ peak between $K_0(1430)$ and $K_2(1430)$. This separation can be made cleanly only by analysing angular correlations between $K^*(1430)$ and the accompanying $K_J(1430)$ ('entanglement'). The result is that the peak is $(75 \pm 3)\%K_0(1430)$ and $25\%K_2(1430)$.

The κ pole optimises at $M = (760 \pm 20(stat) \pm 40(syst)) - i(420 \pm 45 \pm 60)$

MeV. The $K_0(1430)$ is fitted with a Flatté formula including coupling to $K\eta'$; it requires $g^2(K\eta')/g^2(K\eta) = 1.0 \pm 0.3$. A detail is that the Adler zero is also included into the width of the $K_0(1430)$ so that there is an Adler zero in the full $K\pi$ S-wave; this zero improves the fit noticeably.

8 The κ phase

The phase variation of the κ with mass is well determined in two ways which agree. Firstly, there is a large interference between channels $K^*(980)\kappa$ and $KK_1(1270 + 1400)$. Secondly, there is a large interference between κ and $K_0(1430)$, which both contribute to the $K\pi$ S-wave.

A bin-by-bin fit has been made where the κ signal is fitted in magnitude and phase in 10 individual bins 100 MeV wide. Results are shown in Fig. 9 and are in good agreement with the global fit (full curves). [The data points have been adjusted for the phase ϕ_0 of the isobar model, so that the κ phase is zero at threshold]. The agreement between the bin-by-bin fit and the full curve shows that the phase variation can be fitted by the same $D(s)$ in both cases, i.e. purely by a resonance.

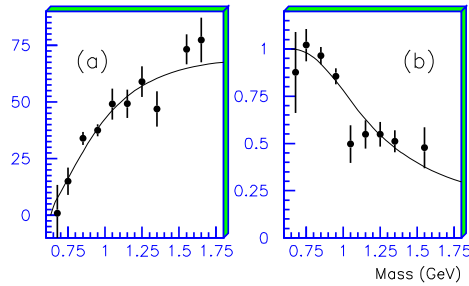


Figure 9: Points show (a) the phase (b) the magnitude of the κ amplitude, determined bin-by-bin; curves shows the global fit.

There is a point of general interest concerning the κ phase. If there is a pole near threshold, why does the observed phase not go through 90° at low mass? This is a subtle point. The clue is that the κ pole lies in the complex s -plane almost below the KK threshold. The factor $(s - s_A)$ in eqn. (8) develops an imaginary part as one goes off the real s -axis. Also the phase space factor $\rho(s)$ becomes complex. The combined effect of these

two effects is to produce a phase rotation of nearly 90° as one moves off the real s -axis to the pole. At the pole, the phase does indeed go through 90° . But on the real s -axis, the constraint of unitarity holds the phase at zero at the $K\pi$ threshold. There is a retardation of the κ phase along the real s -axis of roughly 90° with respect to the pole, although there is also some s -dependence. This was pointed out by Oller [3]; a corresponding smaller effect occurs likewise for the σ pole. On the real s -axis, one is really seeing the upper side of the κ and σ poles but with retarded phases caused by the curious effects of analyticity near threshold. This is an unfamiliar situation and has caused widespread confusion.

A criticism has been made that κ and $K_0(1430)$ cannot be separated in the $K\pi$ S-wave. If so, phases could be adjusted freely in the bin-by-bin fit, but the fit resists that. Secondly, the criticism is logically incorrect. The argument is that magnitudes and phases of both κ and $K_0(1430)$ can be fitted freely as a function of s . If that were true, it would always be impossible to separate any two resonances with the same J^P in a single channel. However, analyticity requires that real and imaginary parts of the amplitude are related and cannot be fitted independently. In the conventional approach, each resonance is fitted by a Breit-Wigner amplitude where magnitude and phase are parametrised by a single function of s . There is indeed flexibility in $N(s)$ for the κ , but only limited flexibility; to a first approximation, the real part of the amplitude is close to the gradient of the imaginary part, as in the simple Breit-Wigner formula. The upshot is that the observed intensity in the $K\pi$ S-wave as a function of s is determined by the intensities of the individual κ and $K_0(1430)$ and the real part of the interference between them. These three terms have distinctively different s -dependence and cannot be confused, except within statistics of each component. Furthermore, interferences of both κ and $K_0(1430)$ with $KK_1(1270 + 1400)$ provide an independent determination of phases. The agreement with the phase in LASS data also seems hardly fortuitous.

9 The BES fit to the κ

An independent analysis of exactly the same data has been reported by the BES group [31]. I do not have the figures, so it is necessary to refer to Ref. [31]. This analysis parametrises the κ in the same general form as

the Ishida model [34]. The $K\pi$ S-wave amplitude is fitted as the sum of a conventional Breit-Wigner amplitude with $\Gamma \propto \rho_{K\pi}$ plus an incoherent background which is adjusted to fit the remaining intensity as a function of mass. The Breit-Wigner amplitude is fitted with $M = 878 \pm 23_{-55}^{+64}$ MeV with Γ at this mass of $499 \pm 45_{-72}^{+48}$ MeV. The corresponding pole position is $M = (841 \pm 30_{-73}^{+81}) - i(309 \pm 45_{-72}^{+48})$ MeV.

There are some important technical differences between the two analyses. In my analysis, one of the phase determinations comes from interference between κ and $K_0(1430)$. Ref. [31] states that the BES analysis omits this interference. Secondly, because of the interference between $K_0(1430)$ and κ , it is important to separate the channels $K^*(890)K_2(1430)$ and $K^*(890)K_0(1430)$. This separation requires analysis of the full angular correlations between decays of $K^*(890)$ and $K_J(1430)$. The BES analysis does not consider $K^*(890)$ decays [31] and therefore separates $K_0(1430)$ and $K_2(1430)$ only from the line-shape. Thirdly, the BES analysis does not include interference between κ and $K_1 \rightarrow \rho K$. For these reasons, the phase of the κ is not determined in that analysis.

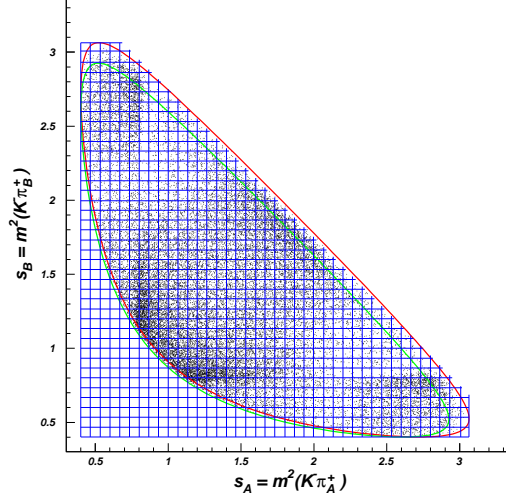


Figure 10: The Dalitz plot for E791 data for $D^+ \rightarrow (K^-\pi^+)\pi^+$.

10 Re-analysis of E791 data

The Dalitz plot for E791 data on $D^+ \rightarrow (K^-\pi^+)\pi^+$ is shown in Fig. 10. Along the $K^*(890)$ bands, there is an obvious asymmetry due to interference with the $K\pi$ S-wave. A new fit has been reported recently where the magnitude and phase of the $K\pi$ S-wave amplitude are fitted separately in 37 mass bins [35]. Results for the magnitude and phase of the $K\pi$ S-wave amplitude will be shown below in Figs. 11 and 12(f).

I have carried out a combined fit to these new E791 data, together with LASS data and BES, with the objective of separating κ and $K_0(1430)$ signals. The BES data define well the $K_0(1430)$ peak, which is much more conspicuous there than in either LASS or E791 data. To a first approximation, the BES data do not determine the κ parameters strongly; they are determined mainly by the LASS and E791 data.

In the E791 fit (and also Ref. [30]), the amplitude for production includes a form factor $F = \exp(-\alpha q^2)$, where q is the momentum of the κ in the D_s rest frame and $\alpha = 2.08 \text{ GeV}^{-2}$. I have varied the exponent α and results are shown in Fig. 11. Panel (a) uses $\alpha = 0$ and achieves the best fit. In (b)–(d) α is increased in equal steps to the E791 value in (d). There is an obvious preference for α close to 0. In fact α optimises at 0 within experimental error for both magnitude and phase. This corresponds to a point-like decay $D_s \rightarrow \kappa\pi$ with an RMS radius $< 0.38 \text{ fm}$ with 95% confidence.

Fits to the BES II data and LASS data are shown in Fig. 12. In BES data, the $K_0(1430)$ is a large signal with well defined centroid and width. The fit to this peak is shown in Fig. 12(e) using 25% $K^*(890)K_2(1430)$ and 75% $K^*(890)K_0(1430)$, as determined from the fit to BES data alone. Figs. 12(c) and (d) show that the fit to the magnitude and phase of the κ in BES data is acceptable. Figs. 12(a) and (b) show the fit to LASS data. There is a small systematic discrepancy around 1.2 GeV which resists a variety of parametrisations. It could be associated with the onset of $KK\pi\pi$ inelasticity which is presently unknown.

From the combined fit, the κ pole position is $M = (750^{+30}_{-55}) - i(342 \pm 60) \text{ MeV}$. This compares with a fit to LASS data alone $M = (722 \pm 60) - i(386 \pm 50) \text{ MeV}$ [36]. This interpretation of E791 data brings results into good agreement with both LASS and BES II data.

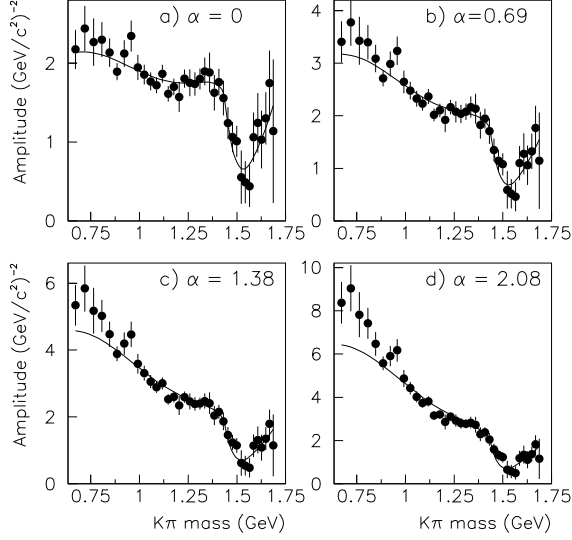


Figure 11: Fits to the magnitude of the κ amplitude in E791 data from Ref. [35], for four values of α in the form factor; α is in units of GeV^{-2} .

11 $f_0(980)$

BES II data on $J/\Psi \rightarrow \phi\pi^+\pi^-$ and ϕK^+K^- both contain prominent $f_0(980)$ signals [37]. The data are fitted with the Flatté formula

$$f = 1/[M^2 - s - i(g_1^2\rho_{\pi\pi} + g_2^2\rho_{KK})]. \quad (9)$$

Fitted parameters are $M = 965 \pm 8(\text{stat}) \pm 6(\text{syst})$ MeV, $g_1^2 = 165 \pm 10 \pm 15$ MeV, $g_2^2/g_1^2 = 4.21 \pm 0.25 \pm 0.21$. There is a second sheet pole at $(998 \pm 4) - i(17 \pm 4)$ MeV, very close to the KK threshold, and a distant third sheet pole at $(851 \pm 28) - i(418 \pm 72)$ MeV. The dominance of the narrow second sheet pole is used by Baru et al. [38] to argue that the $f_0(980)$ is mostly a KK bound state resembling the deuteron.

The best current determination of the parameters of $a_0(980)$ is from Ref. [39]. The second sheet pole is at $M = (1032 \pm 5) - i(85 \pm 9)$ MeV, and the

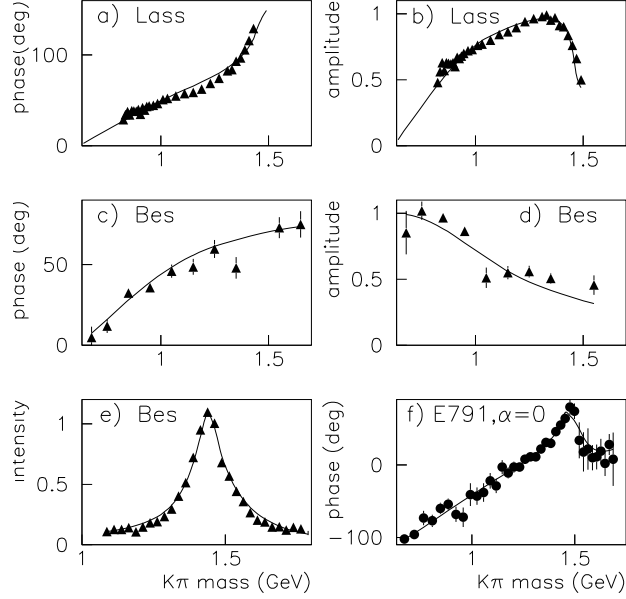


Figure 12: Fits to (a) and (b) the phase and magnitude of LASS amplitudes for elastic scattering, (c) and (d) the phase and magnitude in BES data, (e) the 1430 MeV peak in the $K\pi$ mass projection of BES data, (f) the fit to E791 phases with $\alpha = 0$.

third sheet pole at $M = (981 \pm 8) - i(304 \pm 26)$ MeV. This behaviour is much more like a conventional resonance than $f_0(980)$.

It is often argued that they are anomalously narrow states and therefore decay weakly. This is quite wrong. The coupling of $f_0(980)$ to KK corresponds to a width of 695 MeV when $\rho_{KK} \rightarrow 1$. The resonance appears as a narrow cusp at the KK threshold because the KK channel opens extremely rapidly; the increase in the total width pulls the intensity in the $\pi\pi$ channel down sharply at threshold, making the resonance appear narrow.

There is another interesting way of viewing this effect. The amplitude for

$KK \rightarrow \pi\pi$ is given by

$$f = \frac{1}{k_K} \frac{2g_\pi g_K \sqrt{k_\pi k_K / s}}{M^2 - s - 2i(g_\pi^2 k_\pi + g_K^2 k_K) / \sqrt{s}} \quad (10)$$

where k_π and k_K are centre of mass momenta. The cross section for this process goes as $1/k_K$ near threshold; this is the familiar $1/v$ law for decay to open channels near threshold. The imaginary part of the amplitude follows the optical theorem

$$\text{Im } f = k_K \sigma_{tot} \quad (11)$$

and therefore has a step at threshold. From the dispersion relation for the amplitude

$$\text{Re } f = \frac{1}{\pi} \int \frac{\text{Im } f(s') ds'}{s' - s}, \quad (12)$$

the effect of the step is to generate a narrow peak in the real part at threshold. This is illustrated in Fig. 13 for the scattering length observed for $f_0(980)$. It is a general feature of the opening of any threshold. It provides a mechanism by which a resonance can be attracted to a threshold by the additional attraction of the threshold. It seems likely that this is the explanation of how $f_0(980)$ and $a_0(980)$ are captured by the KK threshold.

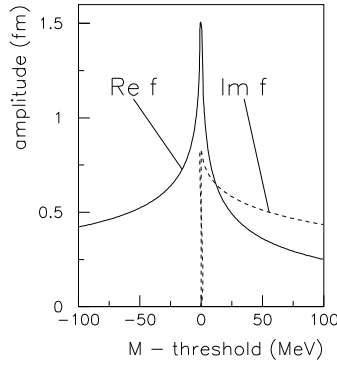


Figure 13: Real and imaginary part of the amplitude for $KK \rightarrow \pi\pi$.

12 What are σ , κ , $f_0(980)$ and $a_0(980)$?

12.1 σ models and Chiral Symmetry breaking

The saga begins in the early 1960's. At this time, it was a puzzle why the lightest mesons were the π and the K , whereas ground-states of even-even nuclei were 0^+ . This led Schwinger to postulate the existence of a σ meson. A second puzzle was the Goldberger-Treiman relation and the fact that the axial weak current is nearly conserved, like the vector current. In an effort to save the Goldberger-Treiman relation from renormalisation effects, Gell-Mann and Levy invented Chiral Symmetry [40] and the linear σ model, placing the pion and the σ on an equal basis. Nambu and Jona-Lasinio invented the non-linear σ model [41]. Today it is still an open question whether these models have any connection with the observed σ pole.

When QCD emerged, it was clear that Chiral Symmetry is broken by the appearance of mass terms of the form $m q \bar{q}$ in the Lagrangian. Today it is almost universally accepted that Chiral Symmetry is spontaneously broken. Spontaneous breaking of a symmetry leads to the existence of a massless Goldstone boson. Many examples are known in condensed matter physics. A classic example is ferromagnetism where, below a critical point, spins align spontaneously; there is an associated massless magnon responsible for spin-waves. In a crystal, the regular spacing of atoms spontaneously creates order; the phonon is the associated massless particle.

It is widely believed that the pion is almost massless because of Chiral Symmetry breaking; this leads to the Adler zero, which has played a prominent role in fitting the data on σ and κ . The small mass of the pion derives from the small masses of u and d quarks. Gell-Mann, Oakes and Renner showed [42] that

$$m_\pi^2 = \frac{m_u + m_d}{f_\pi^2} \left\langle \frac{u\bar{u} + d\bar{d}}{\sqrt{2}} \right\rangle_{0^+}. \quad (13)$$

Here, f_π is the pion decay constant. It is intriguing that the mass of the pion is related to the density of 0^+ fluctuations. It is not presently clear whether these fluctuations have any relation to σ , etc.

A confined quark will surround itself with a cloud of gluons and sea-quarks. Two sets of authors [43,44] suggest that the resulting constituent quarks will acquire a mass of roughly one-third the mass of the nucleon. A σ made of two such constituent quarks would have roughly the right mass.

However a problem with this scheme is that one would then expect the σ to have a brother with $I = 1$ at a very similar mass, whereas the $a_0(980)$ is ~ 400 MeV heavier.

12.2 Jaffe's model

This problem led Jaffe to propose that σ and its relatives are $q^2\bar{q}^2$ states [45]. His suggestion is that there is a pairing interaction forming diquarks in the flavour 3 configuration: ud , ds and us . Then 3 and $\bar{3}$ make a colourless nonet. The σ is the $I = 0$ member $u\bar{d}d\bar{u}$, the κ^+ is $u\bar{s}d\bar{d}$, a_0^0 is $s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$ and $f_0(980)$ is $s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$. This scheme neatly explains why a_0 and f_0 are degenerate in mass and heavier than the σ by twice the mass of the s -quark. It also neatly fits in with the intermediate mass of the κ . Maiani et al. advocate the same scheme [46].

An interesting experimental question is whether there is a further $s\bar{s}s\bar{s}$ state. This is foreign to Jaffe's scheme. GAMS have reported tentative evidence for a narrow state in $\eta\eta'$ at 1914 MeV, almost exactly the $\eta'\eta'$ threshold [47]. Such a state would decay easily to $\eta\eta'$ and would be more prominent there than in $\eta'\eta'$, just as $f_0(980)$ is more prominent in $J/\Psi \rightarrow \phi\pi^+\pi^-$ than in $J/\Psi \rightarrow \phi K^+K^-$: more phase space.

There is support for Jaffe's scheme from recent Lattice QCD calculations of Okiharu et al. [48]. They find configurations of Fig. 14(a) at large radii and those of Fig. 14(b) at small r . Lattice QCD calculations favour the multi-Y-shaped flux-tube configuration for the connected 4-quark system. One can rationalise this scenario with the argument that quarks at large r are non-perturbative and acquire mass from dressing. Another view of this is that it costs energy to establish a flux tube between quarks separated radially. The massive $q^2\bar{q}^2$ configuration can decay by fission to two lighter pions at small r .

There is experimental support for the notion that decays to $\pi\pi$ are at short range. The σ amplitude is known v. momentum k up to nearly 2 GeV/c. Its Fourier transform then determines the radial dependence of the matrix element; the RMS radius of the matrix element is 0.4 fm [36].

There is one important problem with Jaffe's scheme in its simplest version. The ratio $r = g^2(f_0(980) \rightarrow KK)/g^2(a_0(980) \rightarrow KK) = 2.7 \pm 0.5$ disagrees with the ratio 1 predicted from Jaffe's model. [One cannot escape from this problem if $f_0(980)$ and $a_0(980)$ are 2-quark states. Detailed arithmetic on

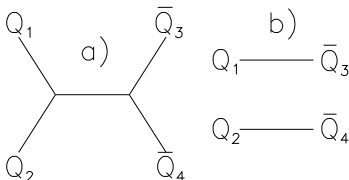


Figure 14: (a) a connected tetraquark configuration, (b) a ‘two-meson’ configuration.

branching ratios, making use of mixing between $f_0(980)$ and σ , predicts a ratio slightly below 1.]

Törnqvist [49] points out that both $a_0(980)$ and $f_0(980)$ *must* contain a KK component in their wave functions. His eqn. (15) includes mesonic channels $A_i B_i$ in loop diagrams:

$$\psi = \frac{|q\bar{q}\rangle + \sum_i [-(d/ds)\text{Re } \Pi_i(s)]^{1/2} |A_i B_i\rangle}{1 - \sum_i (d/ds)\text{Re } \Pi_i(s)}. \quad (14)$$

Here $\text{Re } \Pi(s) = g_{K\bar{K}}^2 \sqrt{4m_K^2/s - 1}$ for $s < 4m_K^2$. [Törnqvist’s equation is written in terms of $q\bar{q}$, but could equally well be reformulated in terms of 4-quark states]. His formula is easily evaluated to find the $K\bar{K}$ components in $a_0(980)$ and $f_0(980)$ as functions of s . At the $K\bar{K}$ threshold, the binding energy $\rightarrow 0$ and the KK wave function extends to infinity, so the KK fraction $\rightarrow 1$. Results are shown in Fig. 15 by the dotted curves. This figure also shows line-shapes as the full curves. The mean KK fraction integrated over the line-shape is $> 60\%$ for $f_0(980)$ and half this for $a_0(980)$.

Suppose decays to $\pi\pi$ and $\pi\eta$ occur at small r , following the Lattice QCD picture. [There will also be some $4q \rightarrow KK$ decay at small r]. When such decays occur, the passive kaonic cloud at large r is left ‘in the air’ (adiabatic approximation), and contributes to ‘fall-apart’ decays. The intensity of these fall-apart decays is proportional to the KK intensity in the wave function, and reduces the discrepancy with Jaffe’s model by a factor 2. However, this argument does not account for $f_0(980)$ lying closer to the KK threshold than $a_0(980)$; that must be taken from experiment.

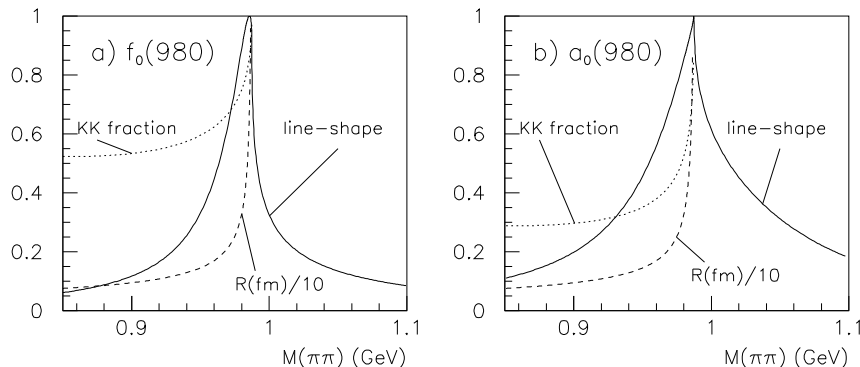


Figure 15: Line shapes of $f_0(980)$ and $a_0(980)$, full curves; the KK fraction in the wave function, dotted curves; $0.1 \times \text{RMS}$ radius, dashed curves.

12.3 CDD poles

There is another important distinction between σ (and its family) and the accepted $q\bar{q}$ states. In $\pi\pi$ elastic scattering, ρ and f_2 exchanges in u and t channels generate attraction. If one takes the K -matrix in the s -channel from these Born terms, the unitarised amplitude $K/(1 - i\rho K)$ reproduces the observed $\pi\pi$ S-wave quite well up to 1 GeV and beyond [50,51]. Similar exchanges are roughly sufficient to generate $f_0(980)$ and $a_0(980)$.

In this respect, σ and its relatives behave completely differently to regular $q\bar{q}$ states like ρ , ω , $K^*(890)$ and ϕ . These cannot be derived from t and u -channel exchanges. It was this fact which led to the quark model. The commonly accepted $q\bar{q}$ resonances appear as CDD poles [52]; these are poles which identically satisfy dispersion relations with no apparent connection with driving terms in $N(s)$ on the left-hand cut.

For this reason, some people view σ , κ , $f_0(980)$ and $a_0(980)$ as molecular states made of mesons. There is, however, an alternative view. In nucleon-nucleon physics, the observed partial waves can be explained in terms of meson exchanges. It is conventional to view these exchanges as generating a potential $V(r)$. In meson physics, it is equally possible to take the view that $\pi\pi$ elastic scattering is telling us about the confining potential itself.

Regular $q\bar{q}$ states are confined within this potential; their leakage out of the potential dictates their decay widths. These widths *must* come out so that they satisfy analyticity over left and right-hand cuts.

Any connection of the σ , κ , $f_0(980)$ and $a_0(980)$ poles to this potential is presently speculative. The formation of nuclei is clearly a phase transition. Chiral Symmetry breaking is also clearly a phase transition; Lattice QCD calculations tell us that the phase boundary is at about 160 MeV. An important question is whether Confinement is the same phase transition as Chiral Symmetry breaking. Present indications from Lattice QCD calculations are that it has a similar transition temperature.

12.4 The scheme of Van Beveren and Rupp

Another interesting, related scheme is that of Van Beveren and Rupp [53-55]. They set out to model the spectrum of all mesons from the lightest to charmonium and bottomonium. This is done with either a harmonic oscillator potential [53], which can be handled algebraically, or any kind of confinement spectrum in a more general approach [54,55]. To allow for decays, they couple $q\bar{q}$ states to outgoing mesons through a transition potential. Ideally, this transition potential has a 3P_0 dependence on radius r [53], but in most calculations it is approximated by a δ function at $r = 0.6\text{--}0.7$ fm or smaller (for heavy $q\text{-}q\bar{q}$ systems). Their equations include a relativistic reduced mass for the two outgoing mesons. This reduced mass generates a zero in the amplitude very similar to the Adler zero.

In this scheme, $f_0(1370)$, $a_0(1450)$, $K_0(1430)$, etc. are regular though unitarised $q\bar{q}$ states; σ , κ , $f_0(980)$ and $a_0(980)$ appear as ‘extra’ states created by coupling of $q\bar{q}$ to mesonic decays. The σ and κ were indeed predicted rather well in 1986 [53]. It is instructive to vary the coupling constant λ between confined states and mesons. For very weak coupling, σ and κ appear as continuum states with infinitely large widths, i.e. at very large $-Im \sqrt{s}$. Fig. 16 shows the analogous $D_s(2317)$ situation. As the coupling between confined $q\bar{q}$ states and outgoing mesons is increased, the continuum state moves towards the real axis; simultaneously, the $c\bar{s}$ state acquires a width and moves off the real s axis. In this way, they account for the $D_s(2317)$ like σ and κ , i.e. as an extra, “molecular” state, created by coupling of the DK continuum to regular $c\bar{s}$ 0^+ state; the latter are pushed to a higher mass than that of the usual funnel potential [55].

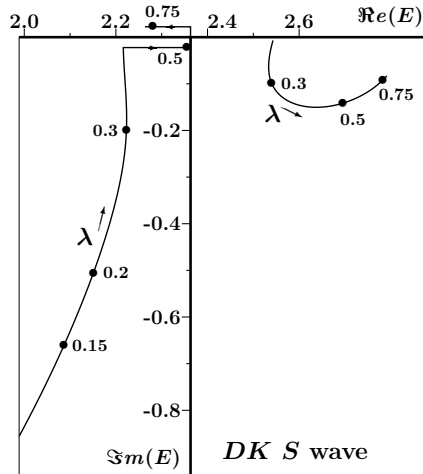


Figure 16: Trajectories as a function of coupling constant λ to decays.

12.5 Extrapolation to $N_c \neq 3$

Theorists have tried varying the numbers of colours using Unitarised Chiral Perturbation Theory [56,57]. They find that as N_c increases, σ and κ fade away into the continuum. [Ref. [58] agrees that they change by large amounts, but reaches different conclusions about the precise change]. This is in contrast to the ρ which survives largely unchanged as N_c increases. That is further evidence that the light scalars have a different character to conventional $q\bar{q}$ states.

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